Restrictions on transverse modes in a parallel-plate waveguide

$\lambda_u, \quad k_u = \frac{2\pi}{\lambda_u}$	Undulator period
K_{rms}	Undulator parameter (rms)
d	Distance between the plates
$k = \frac{\omega}{c} = \sqrt{k_z^2 + k_\perp^2}$	Wave number in vacuum
k_z	Longitudinal wave number in the waveguide
$k_\perp = \frac{n\pi}{d}$	Transverse wave number
$\gamma = \frac{E_e^{kin}}{m_e c^2} + 1$	Electron energy
$\beta_u = \frac{\bar{v}_z}{c}, \quad 1 - \beta_u^2 = \frac{1+K_{rms}^2}{\gamma^2}$	Average longitudinal electron velocity in the undulator
$\Theta = k_z z - \omega t + k_u z$	Ponderomotive phase

k_u has to be replaced by hk_u for higher harmonics h

$$\left(\frac{1-\beta_u^2}{\beta_u^2}\right) = (\gamma_u^2 - 1)^{-1} = (\gamma\beta)^{-2}$$

Resonance condition: $\quad \partial_t\Theta = (k_z + k_u)\beta_u c - kc = 0$

Replacing either k_z by k or k by k_z one gets a quadratic equations with the solutions
+ levert degenerate oplossing / forward wave, dwz $v_g > v_z$

$$k = \frac{\beta_u}{1-\beta_u^2}\left[1 \pm \beta_u\sqrt{1 - \frac{1-\beta_u^2}{\beta_u^2}\frac{k_\perp^2}{k_u^2}}\right]k_u \quad \Rightarrow \quad \frac{\Delta k(n)}{k(0)} = -\frac{1+a^2}{4\gamma^2}\cdot\frac{n^2\lambda_u^2}{4d^2}$$

$$k_z = \frac{\beta_u}{1-\beta_u^2}\left[\beta_u \pm \sqrt{1 - \frac{1-\beta_u^2}{\beta_u^2}\frac{k_\perp^2}{k_u^2}}\right]k_u \qquad \frac{\Delta k(3) - \Delta k(1)}{k_0} = -\frac{1+a^2}{2\gamma^2}\cdot\left(\frac{\lambda_u}{d}\right)^2$$

group velocity $= k_z/k$ $\quad$ force velocity $= k/k_z$

The square root is real for mode numbers

$$n < \frac{2\gamma d}{\sqrt{1+K_{rms}^2}\,\lambda_u} = \sqrt{\frac{2}{\lambda_R\lambda_u}}\,d, \qquad (*)$$

$$= -\frac{\lambda_0\lambda_u}{d^2}$$

where λ_R is the *usual* resonance wavelength

$$= 2.6\,\% \text{ @ } 40\,\mu m$$

$$\lambda_R = \frac{1-\beta_u}{\beta_u}\lambda_u \approx \frac{1+K_{rms}^2}{2\gamma^2}\lambda_u.$$

According to the equation above the electron energy must fulfill

$$\leftarrow \quad \gamma > \frac{n\sqrt{1+K_{rms}^2}\lambda_u}{2d}, \qquad n=1 \rightarrow 5.5 \times \sqrt{(1+K_{rms}^2)}$$

$K_{rms} < 5$

o.k.; $\frac{\Delta k(1)}{k_0} = \frac{-\lambda_0\lambda_u}{8d^2} = -0.2$ voor $\lambda_u = \ldots$, $\lambda_0 = \ldots$

gevoeligheid van variatie in d! $\approx 0.4 \times 8\frac{\Delta d}{d}$ $= 0.4\%$ voor 0.1 mm op 10 m

which is similar to the inequality to have derived.
If the mode number n is larger than the value given in (*) the corresponding mode is not amplified by the electron beam. If the right hand side of (*) is smaller than unity not a single wave can be amplified by the electron beam. If it is larger than unity but smaller than three only a single transverse mode n exists.

zelfs mag ik change z_R
door Gouy phase shift

There exist even tow positive solutions k_z for mode numbers

$$n > \frac{2d}{\lambda_u}$$

and values smaller than (*). In this case, a shorter and a longer wavelength is amplified.
The wave number $k_\perp$ describes waves with a sinuidal shape in the non-wiggle plane. However there also exist transverse modes with different Gauss-Hermite behavior (characterized by the

Faseverschil tussen $n=3$ en $n=1$: over lengte L $\quad \Delta\phi = 2\pi\frac{L\lambda}{d^2} \quad \left(= 2\pi\frac{L[m]\,\lambda[\mu m]}{d^2\,[mm]}\right)$

'gap' afstand [mm] $= \frac{d^2}{\ldots} = \frac{100}{\ldots 2}$ voor FEL 1 mode number m) in the wiggle plane. In contrast to the modes n all the modes m have the same wave vector and hence seem to have the same phase velocity. Due to their Guoy phase, which varies in dependence on m from $-(2m+1)\pi/4$ to $+(2m+1)\pi/4$ within the Rayleigh range, which approximately corresponds to the undulator length, also different modes m have effectively different phase velocities. The additional phase velocity resulting from the Guoy phase is not constant and the case can not be treated as the modes n. A larger phase velocity of the electromagnetic wave means a larger velocity of the ponderomotive wave and shifts the resonance wavelength to larger values. This can be seen in numerical calculations (Kees). The larger m the larger this shift is. If the shift is too large the resonance condition may not be fulfilled and an amplified wave does not exist. The same reduction of modes appears as for the modes n.

The smaller the (vacuum) resonance wavelength is the smaller is the allowed number of transverse modes. Starting from a certain resonance wavelength, which depends on undulator period and waveguide height, an amplified wave does not exist. This upper wavelength bound is much lower than the cut-off wavelength of the corresponding waveguide.