Some notes about Gaussian laser beams applied to FELICE

Joost
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INTENSITY PROFILE OF A GAUSSIAN LASER BEAM

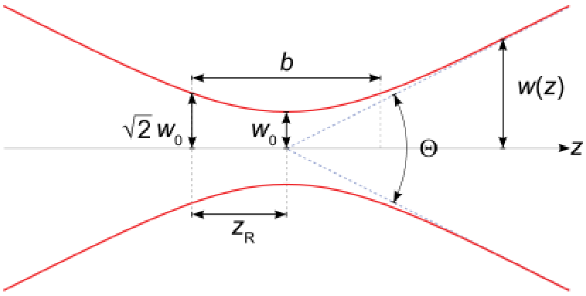

FIG. 1. Picture from Wikipedia.

A Gaussian laser beam is fully characterized by the Rayleigh range z_0, the wavelength λ and the intensity at the center of the beam at the focus position I_0. The radial intensity distribution of the laser running is given by

$$I(x) = I_0 e^{-\frac{2x^2}{w^2}} \tag{1}$$

where w is the waist of the laser, which itself is determined by the distance from the laser focus

$$w(z) = w_0\sqrt{1 + (z/z_0)^2} \tag{2}$$

The waist at the focus position is given by

$$w_0 = \sqrt{z_0\lambda/\pi} \tag{3}$$

For the molecular beam instrument $z_0 = 55$mm so

$$w_0 = 0.13\sqrt{\lambda} \text{ (in mm, } \lambda \text{ in } \mu m) \tag{4}$$

To relate the outcoupled pulse energy to that circulating in the cavity, we have to know the waist of the laser beam at the end mirror, positioned at $z = 1000$mm

$$w(z) = w_0\sqrt{1 + (z/z_0)^2} \tag{5}$$

$$w(1000) = \sqrt{55 \cdot 10^{-2}\lambda/\pi}\sqrt{1 + (1000/55)^2} \tag{6}$$

$$= 2.409\sqrt{\lambda} \text{ (in mm, } \lambda \text{ in } \mu m) \tag{7}$$

RELATION BETWEEN PULSE ENERGY MEASURED AND PULSE ENERGY IN THE CAVITY

The outcouple fraction is the ratio of the integral over the intensity distribution up to the outcoupling hole radius to the intensity distribution integrated over the full beam:

$$f = \frac{\int_0^{d/2} dr r I(r)}{\int_0^{\infty} dr r I(r)} \tag{8}$$

$$= \frac{\int_0^{d/2} dr r \exp\left(-\frac{2r^2}{w^2}\right)}{\int_0^{\infty} dr r \exp\left(-\frac{2r^2}{w^2}\right)} \tag{9}$$

Using $t = \frac{\sqrt{2}r}{w}$; $r = \frac{wt}{\sqrt{2}}$; $dt = \frac{\sqrt{2}dr}{w}$; $dr = \frac{wdt}{\sqrt{2}}$ we simplify this to:

$$f = \frac{\int_0^{\sqrt{2}d/2w} \frac{wdt}{\sqrt{2}}\frac{wt}{\sqrt{2}} e^{-t^2}}{\int_0^{\infty} \frac{wdt}{\sqrt{2}}\frac{wt}{\sqrt{2}} e^{-t^2}} \tag{10}$$

$$= \frac{\frac{w^2}{2}\int_0^{\sqrt{2}d/2w} tdte^{-t^2}}{\frac{w^2}{2}\int_0^{\infty} dtte^{-t^2}} \tag{11}$$

$$= \frac{\int_0^{\sqrt{2}d/2w} tdte^{-t^2}}{\int_0^{\infty} dtte^{-t^2}} \tag{12}$$

The primitive of te^{-t^2} is $-\frac{1}{2}e^{-t^2}$ so this yields:

$$f = \frac{-\frac{1}{2}e^{-t^2}\Big|_0^{\sqrt{2}d/2w}}{-\frac{1}{2}e^{-t^2}\Big|_0^{\infty}} \tag{13}$$

$$= 1 - e^{-\frac{2d^2}{4w^2}} \tag{14}$$

$$\approx \frac{d^2}{2w^2} \tag{15}$$

for $d \ll w$. Combining equations 7 and 15, and using $d = 1$mm we get a numerical value of

$$f = \frac{0.0861}{\lambda} \tag{16}$$

$$= 8.6 \cdot 10^{-6}\tilde{\nu} \tag{17}$$

with $\tilde{\nu}$ in cm^{-1}.

In short, if you measure 1 mJ at 1000 cm^{-1}, the macropulse inside the cavity will be

$$1 \cdot 10^{-3} \cdot \frac{1}{8.6 \cdot 10^{-6} \cdot 1000} = \frac{1}{8.6} = 0.116\text{J} \tag{18}$$

RELATION BETWEEN RELATIVE BANDWIDTH AND PULSE DURATION FOR A GAUSSIAN PULSE

For a transform-limited pulse the time bandwidth product is constant:

$$\Delta\nu\Delta\tau = k \tag{19}$$

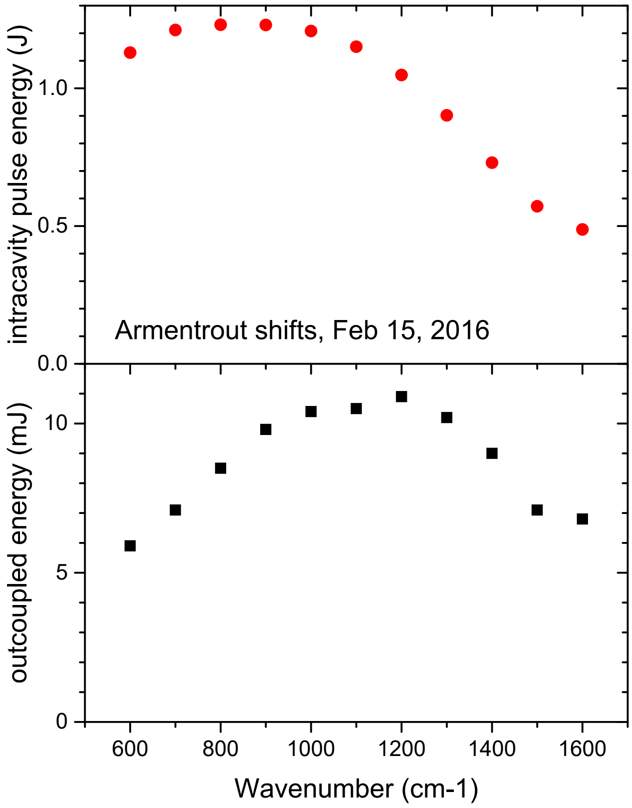

FIG. 2. A sample measurement of outcoupled energy and the inferred macropulse energy within the cavity.

	MB	FTICR
z_0[mm]	55	82
w_0[mm]	$0.13\sqrt{\lambda}$	$0.16\sqrt{\lambda}$
w_{mirr}[mm]	$2.4\sqrt{\lambda}$	$2.0\sqrt{\lambda}$
outcouple fraction	0.0861	0.128

TABLE I. Summary of FELICE parameters

When measured at full-width at half maximum (FWHM) the values for k are 0.44 for a Gaussian pulseand 0.315 for a sech^2-shaped pulse, respectively. To relate the pulse duration to the bandwidth, we use

$$\Delta\tau = \frac{k}{\Delta\nu} \tag{20}$$
$$= \frac{k}{\nu(\Delta\nu/\nu)} \tag{21}$$
$$= \frac{k\lambda}{c(\Delta\lambda/\lambda)} \tag{22}$$
$$= \frac{k\lambda}{c(\Delta\lambda/\lambda)} \tag{23}$$

For a Gaussian pulse, the duration of a pulse is thus:

$$\Delta\tau[fs] = 147\frac{\lambda[\mu m]}{(\Delta\lambda/\lambda)[\%]} \tag{24}$$

The number of optical cycles is simply:

$$N = \nu\Delta\tau \tag{25}$$
$$= \frac{k}{(\Delta\nu/\nu)} \tag{26}$$

Again, for a Gaussian pulse:

$$N = \frac{44}{(\Delta\lambda/\lambda)[\%]} \tag{27}$$

MAXIMUM INTENSITY OF FELICE

The intensity of FELICE micropulses can be estimated from the micropulse energy and the spectral bandwidth. The intensity is given as the ratio of micropulse energy E_μ, laser waist w (equation 2) and micrpulse duration τ_μ

$$I = \frac{E_\mu}{\pi w^2 \tau_\mu} \tag{28}$$
$$= \frac{E_\mu}{\pi\left(\frac{z_0\lambda}{\pi}\left(1+(z/z_0)^2\right)\right)\left(\frac{k\lambda}{c(\Delta\lambda/\lambda)}\right)} \tag{29}$$
$$= \frac{E_\mu c(\Delta\lambda/\lambda)}{kz_0\lambda^2\left(1+(z/z_0)^2\right)} \tag{30}$$

The maximum intensity is found at the focus where $z = 0$, simplying the expression to:

$$I_{MAX} = \frac{E_\mu c(\Delta\lambda/\lambda)}{kz_0\lambda^2} \tag{31}$$

Plugging in the numbers for FELICE ($z_0 = 55$ mm, $E_\mu = 5\cdot10^{-4}$ J at a maximum bandwidth of 12 % (or 5 % RMS)) and assuming a Gaussian pulse shape

$$I_{MAX} = \frac{3\cdot10^{13}}{\lambda[\mu m]^2} \tag{32}$$